MATHEMATICS (Set-1)

Class: XII | Total Marks: 100
Date: | Time: 3 hrs

General Instructions:

(i) All questions are compulsory.
(ii) The question paper consists of 29 questions divided into 3 sections A, B and C. Section-A comprises of 10 questions of 1 mark each, Section-B comprises of 12 questions of 4 marks each and Section-C comprises of 7 questions of 6 marks each.
(iii) All questions in Section-A are to be answered in one word, one sentence or as per the exact requirements of the question.
(iv) There is no overall choice, however internal choice has been given in four questions of 4 marks each and two questions of 6 marks each. You have to attempt only one of the alternatives in all such questions.

SECTION-A

1. Find the value of x and y if $\begin{bmatrix} x+3 & 4 \\ y-4 & x+y \end{bmatrix} = \begin{bmatrix} 5 & 4 \\ 3 & 9 \end{bmatrix}$

2. If $\begin{vmatrix} 3x & 7 \\ 2 & 4 \end{vmatrix} = 10$ then find the value of x.

3. If * be a binary operation defined by a*b = LCM of a and b, find the identity element of * in N.

4. Evaluate: $\int_1^e \frac{\sin(\pi \log x)}{x}$ dx.

5. Write the principal value of $\cos^{-1}\left(\frac{-1}{\sqrt{2}}\right)$.

6. If K is a scalar and A is n rowed square matrix then $|KA|$ equals ------.

7. Evaluate: $\int \frac{e^x}{e^{2x}+1} dx$.

8. Find the area of the parallelogram whose adjacent sides are $\hat{i} - 3\hat{j} + \hat{k}$ *and* $\hat{i} + \hat{j} + 4\hat{k}$.

9. Cartesian equations of a line are 6x-2 = 3y+1 = 2z-2. Find the direction ratios of the line.

10. Find $\vec{a}.\vec{b}$ if $|\vec{a}| = 2 \quad |\vec{b}| = 5, \quad |\vec{a} \times \vec{b}| = 8.$

SECTION-B

11. Find the equation of the normal to the curve $y = \sin^2 x$ at $x = \frac{\pi}{2}$.

12. If $y = \sin^{-1}\frac{2x}{1+x^2} + \sec^{-1}\frac{1+x^2}{1-x^2}$, prove that $\frac{dy}{dx} = \frac{4}{1+x^2}$.

OR

If $y = \sin\ (m\sin^{-1}x), show that (1-x^2)y_2 - xy_1 + m^2 y = 0$.

13. Prove that the relation R in the set A = {1, 2, 3,4, 5} given by R ={(a,b): $|a-b|$ *is even* }, is an equivalence relation.

14. Evaluate: $\int e^{3x}\cos 2x dx.$

OR

Evaluate: $\int \frac{\sqrt{x+3}}{x+2}dx$.

15. If x = 3cos t + cos3t, y = 3 sin t – sin3 t show that $\frac{d^2y}{dx^2} = \frac{\sec^3 t}{6\sin 2t}$.

16. A problem in Mathematics is given to three students A, B and C whose chances of solving it are $\frac{1}{3}, \frac{2}{7} and \frac{3}{8}$ respectively. If all the three try to solve the problem simultaneously find the probability that exactly one of them will solve it.

17. Prove that $\begin{vmatrix} x+a & b & c \\ a & x+b & c \\ a & b & x+c \end{vmatrix} = x^2(x+a+b+c).$

18. Solve the differential equation:
$\cos^2 y \cot x\, dx + \cos^2 x.\ \cot y\, dy = 0.$

19. Solve the differential equation:
$x\frac{dy}{dx} + y = x\log x.$

20. Find the equation of the plane passing through the intersection of the planes
$2x - 3y + z - 4 = 0$ and $x - y + z + 1 = 0$ and perpendicular to the plane $x+2y-3z+6= 0$.

OR

Find the equation of the line passing through (1, 3, 2) and the point of intersection of the line $\frac{x-1}{3} = \frac{y}{2} = \frac{z+1}{7}$ and the plane x + y - z = 8.

21. Prove the following :

$$\cot^{-1}\left(\frac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\right)=\frac{x}{2}, x\in(0,\frac{\pi}{2})$$

OR

Simplify: $\sin^{-1}\left(\frac{\sin x+\cos x}{\sqrt{2}}\right)$, $-\pi/4 < x < \pi/4$

22. Find a unit vector perpendicular to the plane ABC where A, B, C are the points (3, -1, 2); (1, -1, -3); (4, -3, -1), respectively.

SECTION-C

23. Find the co-ordinates of the image of the point (1, 3, 4) in the plane 2x -y + z + 3 = 0.

24. Find the area of the smaller region bounded by the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ and the straight line $\frac{x}{a}+\frac{y}{b}=1$ using integration.

OR

Using integration, find the area of the region bounded by the curves $y = x^2 + 2$, $y = x$, $x = 0$ and $x = 3$.

25. Evaluate: $\int_0^{\pi}\frac{x\tan x}{\sec x+\cos x}dx.$.

26. Using matrix method, solve the following system of equations:

$x - y + z = 4$
$2x + y - 3z = 0$
$x + y + z = 2$

OR

Obtain the inverse of the following matrix using elementary operations:

$$A=\begin{bmatrix}3 & 0 & -1\\ 2 & 3 & 0\\ 0 & 4 & 1\end{bmatrix}.$$

27. A rectangle is inscribed in a semicircle with one of its sides on the diameter of the semicircle. Find the dimensions of the rectangle so that its area is maximum. Find the area also.

28. A manufacturing company makes two models A and B of a product. Each piece of model A requires 9 labour hours for fabricating and 1 labour hour for finishing while each piece of Model B requires 12 labour hours for fabricating and 3 labour hours for finishing. For fabricating and finishing, the maximum labour hours available are 180 and 30 respectively. The company makes a profit of Rs. 8000 on each piece of Model A and Rs. 12,000 on each piece of Model B. How many pieces of Model A and Model B should be manufactured per week to realize a maximum profit? What is the maximum profit per week?

29. A company has two plants to manufacture scooters. Plant-I manufactures 70% of scooters and Plant-II manufactures 30%. At Plant-I, 80% of the scooters are rated standard quality and at Plant-II 90% of scooters are rated standard quality. A scooter is picked up at random and is found to be of standard quality. What is the probability that it has come from Plant-I, Plant-II.